This question paper contains 4 printed pages]

Your Roll No.   : ....

Sl. No. of Q. Paper   : 138   I

Unique Paper Code   : 42351101

Name of the Course   : **B.Sc.(Mathematical Sciences)/B.Sc. (Prog.)**

Name of the Paper   : Calculus and Matrices

Semester   : I

Time : 3 Hours   ***Maximum Marks : 75***

Instructions for Candidates :

  (a) Write your Roll No. on the top immediately on receipt of this question paper.

  (b) Attempt any **Two** questions from each section.

Section - I

1. (a) Prove that the set $\{X_1, X_2\}$ of vectors in R^n is linearly independent iff X_1 and X_2 are collinear.   6

  (b) Define a subspace of a vector space. Examine whether the subset   6

    $W = \{(a,b,2); a.b \in R\}$ of R^3 is a subspace or not.

2. (a) Define Linear Transformation. Find and sketch the image of unit square with vertices $(0,0)$, $(1,0)$, $(0,1)$ and $(1,1)$ under the dilation of factor 3.   6

P.T.O.

(b) Define eighen value of a matrix. Find eighen values and corresponding eighen vectors of matrix   6

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}.$$

3. (a) Find the rank of the matrix

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 3 & 0 & 3 \\ 1 & -2 & -3 & 0 \end{bmatrix} \quad \text{using Elementary}$$

Transformations.

(b) Solve, if consistent, the system of equations :

$x + y + 3z = 1$

$2x + 3y + z = 3$

$5x + 7y + z = 7$ 6, 6

Section - II

4. (a) Discuss the convergence of the following sequences :   6

(i) $\left((-1)^n \cdot \dfrac{1}{n} \right)$ (ii) (x^n)

where $-1 < x < 1$

2

(b) Sketch the graph of the function $f(x) = \frac{1}{2}x^2 - 3x + \frac{11}{2}$.

Mention the transformations used at each step.   6

(c) Radium is known to decay at the rate proportional to the amount present. If half life of radium is 1600 years, what percentage of radium will remain in a given sample after 800 years ?   6

(a) If $y = \sin(m\sin^{-1}x)$, prove that

$$(1-x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2 - m^2)y_n.$$

(b) Find the Taylor's series generated by $f(x) = \frac{1}{x}$

at $x = 2$. When does this series converges to $\frac{1}{x}$.

6

(c) Verify that $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = nz$, where z is given

by $z = (x^2 + xy + y^2)^{-1}$.   6

(a) Find the nth order derivative of the function given by $y = \sin(ax + b)$, where a,b are fixed constants.   6

<div align="center">3</div>

(b) Define heat equation and hence verify that $\varphi(x,t) = e^{-c^2 x^2 t}\sin \pi x$ is a solution of heat equation.

(c) Find the limit of the following sequences

(i) $\dfrac{1^2 + 2 + 3^2 + \ldots\ldots n^2}{6n^3 - n^2 + 3n + 4}$

(ii) $\left(5^{\frac{1}{n}}\right)$

Section - III

7. (a) Show that modulus of sum of two complex numbers is always less than or equal to the sum of their moduli.

   (b) Form an equation in lowest degree with real coefficients which has 2-3i and 3+2i as two of its roots.

8. (a) Solve the equation $z^7 + z = 0$.

   (b) Simplify $\dfrac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5}$.

9. (a) Find the equation of circle whose radius 3 and whose centre has affix 1-i.

   (b) Find the equation of the right bisectors the line joining the points z_1 and z_2.